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MUNDARIJA *** СОДЕРЖАНИЕ *** CONTENTS

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TEKISLIKDA IKKINCHI TARTIBLI CHIZIQLARNING KANONIK KO‘RINISHI

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Annotatsiya. *Ushbu maqolada oliy ta’lim muassasalarida analitik geometriya fanini o‘qitish zaruriyati, ikkinchi tartibli chiziqlarning amaliy masalalarda qo‘llanilish sohalari kengligi, shuning uchun ularni chuqurroq o‘rganishning zarurligi buning uchun esa hosil bo‘lgan tenglamalarni hozirgi zamon axborot texnologiyalaridan foydalangan holda yechish va yechimni grafiklarda tasvirlash talabalar bilimlarini yanada mustahkamlaydi.*

Kalit so‘zlar: *ellips, giperbila, parabola, fokus, ekssentrisitet, direktrisa, ekssentrisitet, direktrisa.*

КАНОНИЧЕСКИЙ ВИД ЛИНИИ ВТОРОГО ПОРЯДКА НА ПЛОСКОСТИ

Аннотация. *В данной статье показана необходимость преподавания науки аналитической геометрии в высших учебных заведениях, широта областей применения линий второго порядка в практических задачах, следовательно, необходимость их углубленного изучения и для этого решения задачи Полученные уравнения с использованием современных информационных технологий и изображением решения в виде графиков еще больше укрепят знания учащихся.*

Ключевые слова: *эллипс, гипербола, парабола, фокус, эксцентриситет, директриса, эксцентриситет, директриса.*

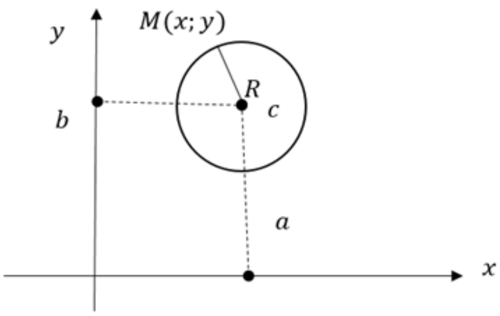
CANONICAL VIEW OF SECOND-ORDER LINES IN THE PLANE

Abstract. *In this article, the need to teach the science of analytical geometry in higher education institutions, the breadth of the fields of application of second-order lines in practical problems, therefore, the need to study them in depth, and for this, solving the resulting equations using modern information technologies and depicting the solution in graphs will further strengthen students' knowledge.*

Keywords: *ellipse, hyperbola, parabola, focus, eccentricity, directrix, eccentricity, directrix.*

Ta’rif. Tekislikda berilgan nuqtadan bir xil uzoqlikdagi nuqtalarning geometrik o‘rniga aylana deyiladi.

Aylananing ta’rifidan foydalanib uning tenglamasini keltirib chiqaramiz. Bizga Dekart koordinatalar sistemasi berilgan bo‘lsin. Koordinatalar sistemasida $C(a;b)$ nuqta berilgan bo‘lsin. $C(a;b)$ nuqtadan bir xil (R) uzoqlikdagi $M(x;y)$ nuqtalar to‘plamiga aylana deyilar ekan.



1-chizma

$|CM| = R$ aylana tenglamasi bo‘ladi.

$|CM| = \sqrt{(x-a)^2 + (y-b)^2}$ ekanligi kelib chiqadi.

$$\sqrt{(x-a)^2+(y-b)^2}=R \Longrightarrow$$
$$(x-a)^2+(y-b)^2=R^2 \qquad (1)$$

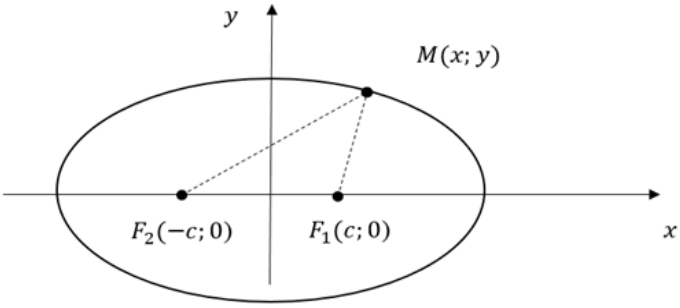
(1) tenglama markazi $C(a;b)$ nuqtada radiusi R ga teng bo‘lgan tenglamasini keltirib chiqardik.

Xususiy hollarda markazi koordinatalar boshida bo‘lgan, ya’ni $O(0;0)$ da bo‘lsa, $x^2+y^2=R^2$ bo‘ladi. Bu tenglama markazi koordinatalar boshida radiusi R ga teng bo‘lgan aylana tenglamasi.

Ta’rif. Tekislikda qo‘zg‘almaydigan ikki nuqtagacha masofalarning yig‘indisi o‘zgarmas bo‘lgan nuqtalarning geometrik o‘rni ***ellips*** deyiladi [1].

Bizga qo‘zg‘almas ikkita nuqta berilgan bo‘lsin. Mana shu qo‘zg‘almas ikki nuqtaga ***fokus*** deyiladi.

Tekislikda ikkita F_1 va F_2 nuqta berilgan bo‘lsin. F_1 va F_2 nuqtalardan to‘g‘ri chiziq o‘tkazamiz va to‘g‘ri chiziqqa yo‘nalish berib uni absissa o‘qi deymiz. F_1 va F_2 nuqtalarning o‘rtasidan ordinata o‘qini o‘tkazamiz.



2-chizma

$|F_1F_2|=2c$ ga teng bo‘lsin, bundan kelib chiqadiki $F_1(c;0)$, $F_2(-c;0)$ bo‘ladi. Ellepsning ta’rifini qanoatlantiruvchi $M(x;y)$ nuqta bo‘lsin.

$$|F_1M|+|F_2M|=2a \text{ bo‘ladi.}$$
$$|F_1M|=\sqrt{(x-c)^2+(y-0)^2},\ |F_2M|=\sqrt{(x+c)^2+(y-0)^2} \Longrightarrow$$
$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$
$$\sqrt{(x-c)^2+y^2}=2a-\sqrt{(x+c)^2+y^2}$$
$$x^2-2xc+c^2+y^2=$$
$$=4a^2-4a\sqrt{(x+c)^2+y^2}+x^2+2xc+c^2+y^2$$
$$-2xc=4a^2-4a\sqrt{(x+c)^2+y^2}+2xc$$
$$4a^2-4a\sqrt{(x+c)^2+y^2}+4xc=0$$
$$a^2-a\sqrt{(x+c)^2+y^2}+xc=0$$
$$a^2+xc=a\sqrt{(x+c)^2+y^2}$$
$$a^4+2a^2xc+x^2c^2=a^2(x^2+2xc+c^2+y^2)$$
$$a^4+2a^2xc+x^2c^2=a^2x^2+2a^2xc+a^2c^2+a^2y^2$$
$$a^4+x^2c^2=a^2x^2+a^2c^2+a^2y^2$$
$$a^4+x^2c^2-a^2x^2-a^2c^2-a^2y^2=0$$
$$a^4+x^2c^2-a^2x^2-a^2c^2-a^2y^2=0$$
$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$
$$a^2-c^2=b^2 \Longrightarrow b^2x^2+a^2y^2=a^2b^2 \Longrightarrow$$
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (2)$$

ekanligi kelib chiqadi.

Fokuslar orasidagi masofani katta o‘qqa nisbatiga ekssentrisitet deyiladi.

$$\varepsilon=\frac{2c}{2a} \Longrightarrow \varepsilon=\frac{c}{a} \qquad (3)$$

Ellipsning kichik o‘qiga parallel va uning markazidan a/ε masofadan o‘tuvchi parallel to‘g‘ri chiziqlar ellipsning ***direktrisalari*** deyiladi.

$$x=\pm\frac{a}{\varepsilon}=\pm\frac{a}{c/a}=\pm\frac{a^2}{c}$$

1-Misol. $x^2+4y^2=4$ tenglama ellipsni ifodalashini ko‘rsating va uning barcha xarakteristikalarini toping.

Yechish: Dastlab berilgan tenglamani ikkala tomonini 4 soniga bo‘lamiz:

$$\frac{x^2}{4}+\frac{y^2}{1}=1$$

bu yerdan berilgan tenglama yarim o‘qlari $a=2$ va $b=1$ bo‘lgan ellipsni ifodalashini ko‘ramiz. Unda $c^2=a^2-b^2=3$ bo‘lgani uchun qaralayotgan ellipsning fokuslari $F_1(\sqrt{3};0)$ va $F_2(-\sqrt{3};0)$ nuqtalarda joylashganligini ko‘ramiz. Bu natijalardan foydalanib, ellipsning ekssentrisiteti va direktrisalarini topamiz:

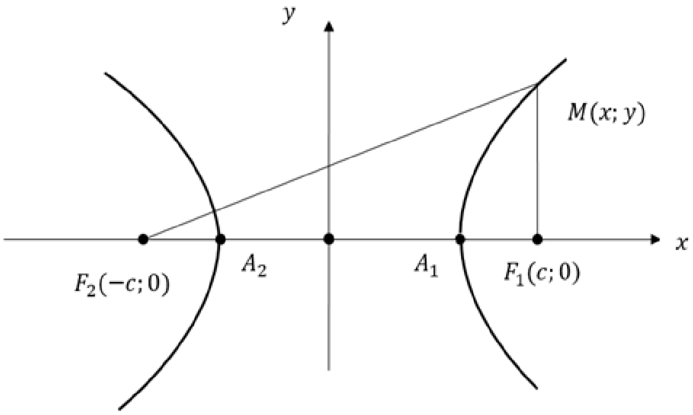
$$\varepsilon=\frac{c}{a}=\frac{\sqrt{3}}{2},\quad x=\pm\frac{a}{\varepsilon}=\pm 2:\frac{\sqrt{3}}{2}=\pm\frac{4\sqrt{3}}{3}.$$

Ellipsga tegishli $M(x;y)$ nuqtaning fokal radiuslari

$$r_1=a+\varepsilon x=2+\frac{\sqrt{3}}{2}x,\quad r_2=a-\varepsilon x=2-\frac{\sqrt{3}}{2}x$$

formulalar bilan topiladi.

Giperbola. Fokus deb ataladigan ikki nuqtagacha bo‘lgan masofalarining ayirmasi o‘zgarmas songa teng bo‘lgan nuqtalarning geometrik o‘rniga ***giperbola*** deyiladi. Fokuslar - F_1, F_2, ular orasidagi masofa $|F_1F_2|=2c$. Fokuslar yotgan to‘g‘ri chiziqqa yo‘nalish berib absissa o‘qi deylik.



3-chizma

Absissa o‘qini 2 ta fokusdan o‘tkazaylik. Fokuslarning o‘rtasidan absissa o‘qiga perpendikulyar qilib ***ordinata*** o‘qini o‘tkazaylik.

$$|F_1M|-|F_2M|=2a$$

$$|F_1M|=\sqrt{(x-c)^2+y^2},\ |F_2M|=\sqrt{(x+c)^2+y^2}$$

$$\left|\sqrt{(x-c)^2+y^2}-\sqrt{(x+c)^2+y^2}\right|=2a$$

$$\sqrt{(x-c)^2+y^2}=\pm 2a+\sqrt{(x+c)^2+y^2}$$

$$x^2-2xc+c^2+y^2=x^2+2xc+c^2+y^2\pm 4a\sqrt{(x+c)^2+y^2}+4a^2$$

$$\pm 4a\sqrt{x^2+2xc+c^2+y^2}=4a^2+4xc$$

$$a^2x^2+2a^2xc+a^2c^2+a^2y^2=a^4+2a^2xc+x^2c^2$$

$$x^2(a^2-c^2)-a^2(a^2-c^2)+a^2y^2=0$$

$$x^2(a^2-c^2)-a^2y^2=a^2(a^2-c^2)\Longrightarrow c>0\Longrightarrow\ c^2-a^2=b^2$$

$$\Delta MF_1F_2\Longrightarrow\ F_1M-F_2M=2a\ \Longrightarrow\ |F_1M-F_2M|<|F_1F_2|\Longrightarrow$$

$$2a<2c\Longrightarrow a<c$$

$$x^2b^2-a^2y^2=a^2b^2\Longrightarrow$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \tag{4}$$

ekanligi kelib chiqadi.

Bu yerda $2a$ −haqiqiy o‘q, $2b-$ mavhum o‘q, $2c-$ fokuslar orasidagi masofa.

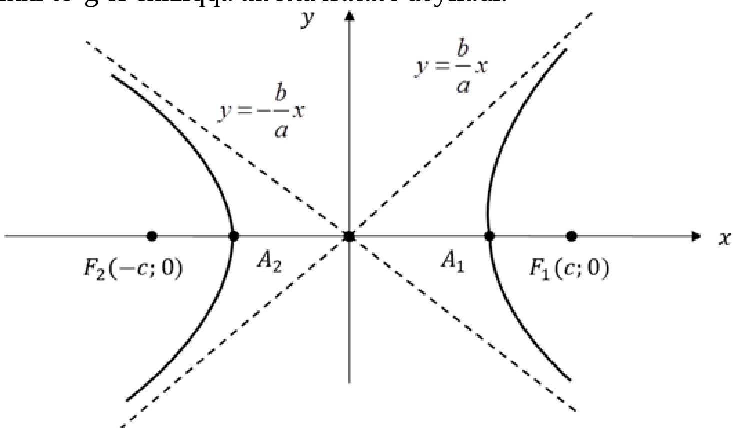
Giperbolaning ekssentrisiteti deb, giperbola fokuslari orasidagi masofaning haqiqiy o‘qqa nisbatiga aytiladi [2].

$$\varepsilon=\frac{2c}{2a}=\frac{c}{a}\ \Longrightarrow c^2-a^2=b^2\Longrightarrow c>a\ \Longrightarrow\varepsilon>1$$

Giperbolaning mavhum o‘qiga parallel va uning markazidan

$$x = \pm\frac{a}{\varepsilon} = \pm\frac{a}{c/a} = \pm\frac{a^2}{c}$$

masofada yotuvchi ikki to‘g‘ri chiziqqa ***direktrisalari*** deyiladi.



4-chizma

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Longrightarrow d{:}\, x = \pm\frac{a}{\varepsilon} = \pm\frac{a^2}{c}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Longrightarrow \ a < c.$$

$$y = \pm\frac{b}{a}x \qquad (5)$$

asimptota deyiladi.

$a = b$ bo‘lsa, ***teng yonli geperbola*** deyiladi.

Fokal radiuslar. Ellips uchun

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ \Longrightarrow \ \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \ \Longrightarrow$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right) \Longrightarrow a^2 - c^2 = b^2$$

$$|F_1M| = r_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{(x-c)^2 + b^2 - \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2x^2}{a^2}} = \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{\frac{a^2 - b^2}{a^2}x^2 - 2xc + c^2 + b^2} = \sqrt{\frac{c^2}{a^2}x^2 - 2xc + a^2} =$$

$$= \left|\frac{c}{a}x - a\right| = |\varepsilon x - a|.$$

$$|F_2M| = r_2 = \sqrt{(x+c)^2 + y^2} = \left|\frac{c}{a}x + a\right| = |\varepsilon x + a| \ \Longrightarrow$$

$$0 < c < a$$

$$r_1 = a - \frac{c}{a}x, \quad r_2 = a + \frac{c}{a}x \ \ \Longrightarrow r_1 + r_2 = 2a$$

Ellips yoki giperbola uchun fokal radiusi degani, uning biror nuqtasidan fokuslarigacha bo‘lgan masofalar.

Giperbola uchun

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Longrightarrow \frac{x^2}{a^2} - 1 = \frac{y^2}{b^2} \ \Longrightarrow y^2 = b^2\left(\frac{x^2}{a^2} - 1\right)$$

$$|MF_1| = r_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{x^2 - 2xc + c^2 + \left(\frac{x^2}{a^2} - 1\right)b^2} =$$

$$= \sqrt{x^2 - 2xc + c^2 - b^2 + \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{\frac{(a^2+b^2)}{a^2}x^2 - 2xc + a^2 + b^2 - b^2} = \left|\frac{c}{a}x - a\right|.$$

$$|MF_2| = r_2 = \sqrt{(x+c)^2 + y^2} = \left|\frac{c}{a}x + a\right| \ \Rightarrow c^2 - a^2 = b^2 \Rightarrow$$

$$c < a < 0$$

$$r_1 = \left|\frac{c}{a}x - a\right| = \frac{c}{a}x - a, \quad r_2 = \frac{c}{a}x + a \qquad (6)$$

bo‘ladi.

2-Misol. Quyidagi kanonik tenglamasi bilan berilgan giperbolaning barcha xarakteristikalarini toping:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Bu giperbolaning absissasi $x = 8$, ordinatasi $y > 0$ bo‘lgan M nuqtasining fokal radiuslarini aniqlang.

Yechish: Berilgan tenglamani (4) kanonik tenglama bilan taqqoslab, giperbolaning haqiqiy va mavhum yarim o‘qlari $a = 4$, $b = 3$ ekanligini ko‘ramiz.Bu holda $c^2 = a^2 + b^2 = 16 + 9 = 25 \ \Rightarrow c = 5$ bo‘lgani uchun giperbolaning fokuslari $F_1(5;0)$ va $F_2(-5;0)$ nuqtalarda joylashganligini aniqlaymiz. Berilgan giperbolaning asimptotalari

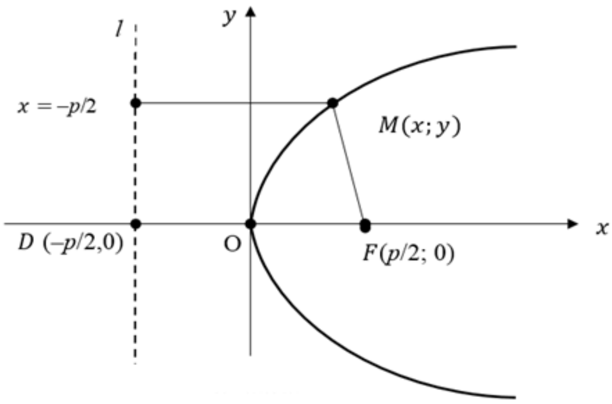
$$y = \pm\frac{b}{a}x = \pm\frac{3}{4}x = \pm 0{,}75x$$

ekssentrisiteti $\varepsilon = c/a = 5/4 = 1{,}25$, direktrisalarining tenglamasi esa $x = \pm a/\varepsilon = \pm 4/1{,}25 = \pm 3{,}2$ bo‘ladi. Endi giperbolaning berilgan $M(8;0)$ nuqtasining fokal radiuslarini topamiz. Bu nuqta giperbolaning o‘ng shoxida joylashgan va shu sababli (6) formulani $"+"$ ishora bilan qaraymiz:

$$r_1 = a + \varepsilon x = 4 + 1{,}25 \cdot 8 = 14,$$
$$r_2 = -a + \varepsilon x = -4 + 1{,}25 \cdot 8 = 6$$

Parabola. Berilgan nuqtadan va berilgan to‘g‘ri chiziqdan bir xil uzoqlikda joylashgan nuqtalarning geometrik o‘rniga ***parabola*** deyiladi.

Ta’rifdan foydalanib, parabolaning kanonik tenglamasini keltirib chiqaraylik. Bizga F nuqta va l to‘g‘ri chiziq berilgan ekan. F nuqtadan o‘tib l to‘g‘ri chiziqqa perpendikulyar qilib Ox o‘qini olaylik. l to‘g‘ri chiziqqa parallel va F nuqta bilan l to‘g‘ri chiziqni o‘rtasidan Oy o‘qini o‘tkazaylik. Kanonik tenglamasini topmoqchi bo‘lgan parabola ustidan ixtiyoriy $M(x;y)$ nuqta olaylik. F nuqtadan l to‘g‘ri chiziqqacha bo‘lgan masofa P bo‘lsin. U holda F nuqtaning koordinatalari $F(\frac{p}{2};0)$ va l to‘g‘ri chiziqning tenglamasi $x = -\frac{p}{2}$ bo‘ladi. F nuqtadan M nuqtagacha bo‘lgan masofa $|FM| = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}$ bo‘ladi. M nuqtadan l to‘g‘ri chiziqqacha bo‘lgan masofa esa $d = \left|x + \frac{p}{2}\right|$ bo‘ladi [3].



5-chizma

$$|FM| = |Fd|$$

$$\sqrt{\left(x - \frac{p}{2}\right)^2 + y^2} = \left|x + \frac{p}{2}\right| \Rightarrow x^2 - 2x\frac{p}{2} + \frac{p^2}{4} + y^2 =$$

$$= x^2 + 2x\frac{p}{2} + \frac{p^2}{4} \Rightarrow y^2 - xp = xp \Rightarrow$$

$$y^2 = 2px \qquad (7)$$

tenglamamiz ***parabola tenglamasi*** hisoblanadi.

3-Misol. Ox o‘qi parabolaning simmetriya o‘qi bo‘lib, uning uchi koordinatalar boshida yotadi. Parabola uchidan fokusigacha bo‘lgan masofa 4 birlikka teng. Parabola va uning direktrisasi tenglamasini toping.

Yechish: Dastlab, masala shartiga asosan, parabolaning p parametrini topamiz:

$$|OF| = 4 \Rightarrow p/2 = 4 \Rightarrow p = 8.$$

Unda, (7) formulaga asosan, parabola tenglamasini topamiz:

$$y^2 = 2px \Rightarrow y^2 = 2 \cdot 8x = 16x.$$

Bu yerdan direktrisa tenglamasi $x = -p/2 \Rightarrow x = -4$ ekanligini ko‘ramiz.

Shuni ta’kidlab o‘tish kerakki, $y = ax^2 + bx + c$ $(a \neq 0)$ kvadrat uchhadning grafigi uchi koordinatalari

$$x_0 = -\frac{b}{2a}, \quad y_0 = \frac{4ac - b^2}{4a}$$

bo‘lgan $M_0(x_0; y_0)$ nuqtada, simmetriya o‘qi esa Oy o‘qiga parallel va $x = -b/2a$ tenglamaga ega bo‘lgan vertikal to‘g‘ri chiziqdan tashkil topgan paraboladan iboratdir. Agar $a > 0$ bo‘lsa, parabola yuqoriga, $a < 0$ bo‘lsa, pastga yo‘nalgan bo‘ladi.

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