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Solving Equations Using Polar Coordinates

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Abstract: This work is dedicated to solving some equations of elementary mathematics using the polar coordinate system. In mathematics, solving some equations in the usual way causes us a number of problems, and solving such equations using the polar coordinate system helps us find the answer much easier. The article first gives information about the polar coordinate system and shows several examples.

Keywords: Polar Coordinate System, Generalized Polar Coordinate System, Scale, Polar Axis

Introduction

As we know, to determine the position of a point, a Cartesian coordinate system with right angles is used. In addition to this system, there are several other coordinate systems.

For example: One of them is the polar coordinate system, which greatly simplifies the solution of many problems and examples.

Using this system, the position of a point on a plane is determined by specifying a point called a pole, a ray called the polar axis extending from that point, and a scale to measure the length. In addition, when specifying the polar system, you should also indicate which rotations around the point are considered positive. Generally, counterclockwise turns are considered positive.

If the polar angle moves counterclockwise from the polar axis, it is considered positive, and if clockwise, it is considered negative. we agree to understand an angle as an angle in trigonometry, that is, we look at this angle up to its sign. and are called the polar coordinates of the point and are written on the drawing. the value of the polar angle of a point satisfying the inequalities is called the principal value of its polar angle.

Methods

The research method used in solving the problem includes an analytical method. Figure captions should be part of the figure caption, not part of the figure. The methods used in the completion of the research are written in this section. In the Research Methods, small and non-main tools (already common in the lab, such as: scissors, measuring cups, pencils) do not need to be written down, but simply write down the main set of equipment only, or the main tools used for analysis and / or characterization, even need to go to type and accuracy; Write in full the location of the research, the

number of respondents, how to process the results of observations or interviews or questionnaires, how to measure performance benchmarks; common methods do not need to be written in detail, but simply refer to the reference book. Experimental procedures should be written in the form of news sentences, not command sentences.
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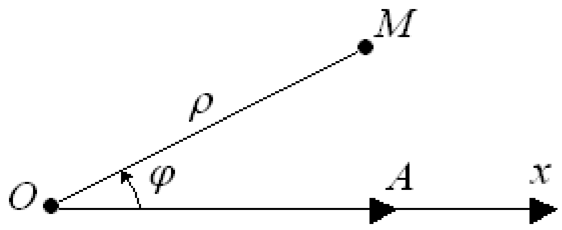
Results and Discussion

According to the above, the inequality is valid for polar coordinates. You can also not put such a condition on polar coordinates. In this case, when , it is considered that it has changed from to. When the angle changes from to, the polar coordinate system is called a generalized polar coordinate system.



1-picture

Graph of the polar coordinate system.

In the polar coordinate system, the position of any point M on the plane is determined by the distance between this point and the pole head O and the angle between OM and the polar axis OA.

is called the polar radius of point M, and the angle φ is called the polar angle of point M. Taking into account the sign of the angle, an accuracy of up to can be obtained. Thus, the polar coordinates of point M are written in the form . Among the possible values of the polar angle of point M, select the exact value that satisfies the inequality , and call this value the main value

we call it. If the polar coordinates r and φ vary from -∞ to +∞, the polar coordinate system is called a generalized polar coordinate system.

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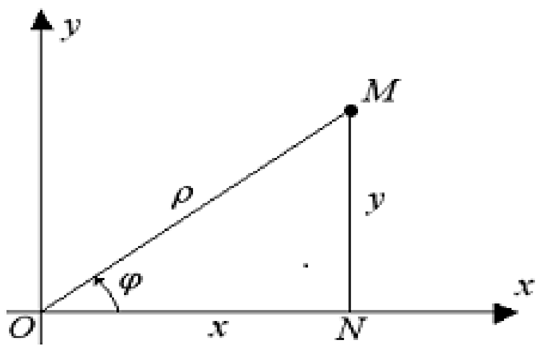


2-picture

Cartesian with polar coordinate system

connection graph of the coordinate system

Now let's consider the problem that if the polar coordinates of a point are known, its Cartesian coordinates should be calculated, and vice versa, if the Cartesian coordinates of a point are known, its polar coordinates should be calculated.

Let's say that the pole of the polar coordinate system coincides with the origin of the rectangular Cartesian coordinate system, and the polar axis coincides with the positive half of the abscissas (2-picture). Let M be an arbitrary point of the plane, let be its Cartesian coordinates, and let be its polar coordinates.

There is an

$$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases} \qquad (1)$$

relationship between these coordinates. Based on these formulas:

$$x^2 + y^2 = \rho^2 \qquad (2)$$

$$\varphi = arctg\,\frac{y}{x} \qquad (3)$$

relations can be written.

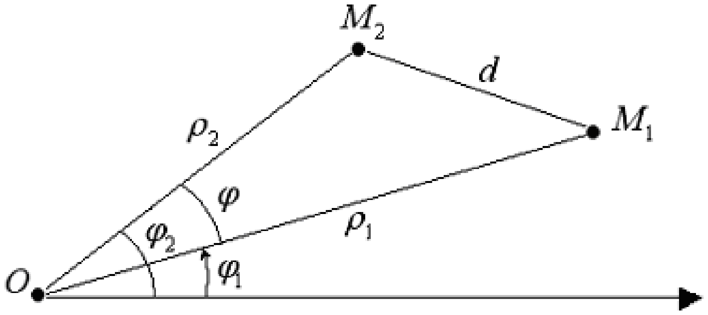
Problem-1. Find the distance d between points and in the polar coordinate system.

Solution: Based on the theorem of cosines:

$$d^2 = OM_1^2 + OM_2^2 - 2OM_1 \cdot OM_2 \cdot \cos\varphi \quad \text{in this } \varphi = \varphi_2 - \varphi_1$$

$$d^2 = \rho_1^2 + \rho_2^2 - 2\rho_1 \cdot \rho_2 \cdot \cos(\varphi_2 - \varphi_1) \quad \text{or} \quad d = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1 \cdot \rho_2 \cdot \cos(\varphi_2 - \varphi_1)}$$

will be (*picture 3*).



picture 3

Points M1 and M2 in the polar coordinate system graph of the distance between

Problem-2: This

$$\sqrt[3]{3x+5}-\sqrt{x+3}=0$$

solve the equation

Solution: To solve this equation using the polar coordinate system

$$\sqrt[3]{3x+5}=r\sin\beta\ ,\ \sqrt{x+3}=r\cos\beta$$

let's define it as

$$\begin{cases} 3x+5=r^3\sin^3\beta \\ 3x+9=3r^2\cos^2\beta \end{cases}$$

$$\begin{cases} r\sin\beta-r\cos\beta=0 \\ r^3\sin^3\beta-3r^2\cos^2\beta=-4 \end{cases}$$

$$r\sin\beta= r\cos\beta,\ \ \beta=\frac{\pi}{4}+2\pi k,\ \ r\sin\beta=t$$

$$3r^2\cos^2\beta-r^3\sin^3\beta=4$$

$$-t^3+3t^2-4=0$$

$$t^3-3t^2+4=0$$

$$t^3+t^2-4t^2+4=0$$

$$t^2(t+1)-4(t^2-1)=0$$

$$(t+1)(t^2-4t+4)=0$$

$$r\sin\beta=2;\ \ r\sin\frac{\pi}{4}=2\ \ \ r=\frac{2}{\frac{\sqrt{2}}{2}}=\frac{4}{\sqrt{2}}$$

$t+1=0$, $t_1=-1$ cannot be a root

$$t^2-4t+4=(t-2)^2$$

$$X+3\left(\frac{4}{\sqrt{2}}\right)^2*\left(\frac{\sqrt{2}}{2}\right)^2;\ \ \ x+3=\frac{16}{2}*\frac{2}{4}=4$$

$$X=4-3=1$$

Answer: x=1

***<u>Problem-3</u>*:**

$$\sqrt{1+\sqrt{x}}+\sqrt{1-\sqrt{x}}=2$$

solve the equation.

Solution: The domain of definition of Eq

$$\begin{cases} 1+\sqrt{x}\geq 0 \\ 1-\sqrt{x}\geq 0 \end{cases};\ x\geq 0; x\leq 1$$

We introduce a new variable to solve the equation

$$\begin{cases}\sqrt{1+\sqrt{x}} = r\,sina \\ \sqrt{1-\sqrt{x}} = r\,cosa\end{cases} \Leftrightarrow \begin{cases}1+\sqrt{x} = r^2sin^2a \\ 1-\sqrt{x} = r^2cos^2a\end{cases}$$

$$2=r^2sin^2a+r^2cos^2a$$

$$\begin{cases} r\,sina + r\,cosa = 2 \\ r^2sin^2a+r^2cos^2a = 2\end{cases} \Leftrightarrow \begin{cases} r(\,sina +\ cosa) = 2 \\ r^2(sin^2a + cos^2a) = 2\end{cases} \Leftrightarrow \begin{cases} r(\,sina +\ cosa) = 2 \\ r^2 = 2\end{cases}$$

$sin^2a + cos^2a = 1$ from being

$$Cosa+sina=\sqrt{2}\sin(\frac{\pi}{4}+a)$$

$$=r\sqrt{2}\sin\left(\frac{\pi}{4}+a\right) = 2$$

$$r^2 = 2$$

$r_1=\sqrt{2}$; $r_2=-\sqrt{2}$ cannot be a root

$$\sqrt{2}\sqrt{2}\sin\left(\frac{\pi}{4}+a\right) = 2 \Leftrightarrow \sin\left(\frac{\pi}{4}+a\right) =1 \Leftrightarrow \frac{\pi}{4}+a = \frac{\pi}{2}+2k\pi \Leftrightarrow$$

$$a=\frac{\pi}{4}+2k\pi.$$

$$\sqrt{1+\sqrt{x}}=\sqrt{2}sin\frac{\pi}{4},\ 1+\sqrt{x} = 2sin^2\frac{\pi}{4}$$

$$\sqrt{x} = 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1;\quad x=0$$

Answer: x=0

Problem-4:

$$\sqrt{1+x}+\sqrt{1-x} = 2$$

solve the equation:

Solution: We know that the domain of Eq

$$\begin{cases}1+x \geq 0 \\ 1-x \geq 0\end{cases};\ x \geq -1;\ x \leq 1$$

we introduce a new variable to solve the equation.

$$\sqrt{1+x}=r\,sina\,;\quad 1+x=r^2sin^2a$$

$$\sqrt{1-x}=r\,cosa;\quad 1-x=r^2cos^2a;$$

$$\begin{cases} r\sqrt{2}sin\left(\frac{□}{4}+a\right) = 2 \\ r^2(sin^2a+cos^2a) = 2\end{cases} => \begin{cases} r\sqrt{2}sin\left(\frac{□}{4}+a\right) = 2 \\ r^2 = 2\end{cases} =>$$

$r_1 = \sqrt{2}$; $r_2 = -\sqrt{2}$ cannot be a root $\quad r = \sqrt{2}\sqrt{2}\,sin(45^o+a) = 2;$

$$sin(45^o+a) = 1;\quad a=\frac{\pi}{2}+2\pi k-\frac{\pi}{4}$$

$$a = 2\pi k + \frac{\pi}{4}$$

$$1+x = \left(\sqrt{2}\right)^2 \sin^2\frac{\pi}{4}$$

$$X+1 = 2\left(\frac{\sqrt{2}}{2}\right)^2; \quad x=0.$$

Answer: x=0

Problem-5:

$$\sqrt{7x+9} + \sqrt{9-7x} = 6$$

solve the equation:

Solution: $\sqrt{7x+9} = rsina$; $\sqrt{9-7x} = rcosa$

$$\begin{cases} r\,sina + rcosa = 6 \\ r^2 sin^2 a + r^2 cos^2 a = 18 \end{cases} \Rightarrow$$

$$r^2(sin^2 a + cos^2 a) = 18; \quad r^2 = 18$$

$r_1=3\sqrt{2}$; $r_2=-3\sqrt{3}$ cannot be a root

$$r\,(sina + cosa) = 6 \quad r\sqrt{2}\sin\left(\frac{\pi}{4} + a\right) = 6$$

$$3\sqrt{2}\sqrt{2}\sin\left(\frac{\pi}{4} + a\right) = 6$$

$$\sin\left(\frac{\pi}{4} + a\right) = \frac{6}{3\sqrt{2}\sqrt{2}} = 1$$

$$\frac{\pi}{4} + a = 2\pi k + \frac{\pi}{2}; \quad a = 2\pi k + \frac{\pi}{4}$$

$$\sqrt{7x+9} = \left(3\sqrt{2}\right)^2 sin^2\frac{\pi}{4}$$

$$7x+9 = 9\cdot 2\cdot \frac{\sqrt{2}}{4}$$

$$7x+9=9 \quad x=0$$

Answer: x=0

Conclusion

The exploration of solving elementary mathematical equations using the polar coordinate system reveals significant advantages in terms of simplification and efficiency. This study illustrated how complex equations, which pose substantial challenges in Cartesian coordinates, can be effectively tackled by converting them into polar form. This approach not only provides alternative methods for solving but also enhances understanding of the underlying geometric interpretations of equations. The findings underscore the potential of polar coordinates in addressing a broader range of mathematical problems, thereby suggesting the need for further research into its applications across different branches of mathematics and physics. Future studies could focus on developing more advanced techniques and identifying new areas where the polar coordinate system could be particularly beneficial, thus expanding its utility and application in both theoretical and practical contexts.

References

[1] A.G.Kurosh Oliy algebra kursi. Toshkent “O’qituvchi” 1976
[2] Uzoqboyev, A., Abdullayev, S., & Abriyev, N. (2023). Robototexnik mexanizmlarning maxsusliklarini izlashda matritsaviy usulning qo’llanishi. Евразийский журнал математической теории и компьютерных наук, 3(1), 92-100.
[3] Абрамович М.А. Стародубстев М.А. “Математика”. Тошкент. Укитувчи 1985
[4] Abduhamidov A.U, Nasimov H.A,Nosirov U.M, Husanov Z.H.“Algebra va matematik analiz asoslari”. O’qituvchi. Nashriyot-matbaa ijodiy uyi. Toshkent 2008
[5] Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberdiyev, F.H.Haydarov Algebra va sonlar nazariyasi (o’quv qo’llanma). Toshkent. “Tafakkur-bo’stoni” 2019.
[6] Брюно А.Д. Солеев А. Локальная униформизация ветвей пространственной кревой и многоранники Ньютона∕∕Алгебра ианализ Т. 3, вып. 1, (1991), С. 67-102.
[7] A.Soleyev, X.Nosirova. Darajali geometriyaning chiziqli bo’lmagan masalalarga qo’llanilishi. Monografiya . Samarqand: SamDU, 2017.
[8] Saxayev M. “Elementar matematika masalalari to’plami” I.II qismlar.”O’qituvchi” Toshkent 1970.1972. 220-236 bet.
[9] Брюно А.Д. Солеев А. Локальная униформизация ветвей пространственной кревой и многоранники Ньютона∕∕Алгебра ианализ Т. 3, вып. 1, (1991), С. 67-102.
[10] Сборник конкурсних задач по математике (под редак М.И.Сканави) « Вышая школа» М 1978. 107-122 стр.